Algorithmic trading using Artificial Intelligence tools.

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Abstract

Given two cointegrated stocks, we construct a convenient synthetic asset and go on to capture statistical arbitrage opportunities in the pair of stocks by using a feedforward neural network to build a model for the dynamics of the distortion process (mispricing) of the synthetic asset in wich the volatility of its residuals follows a GARCH process. Forecasts for the mispricings are represented as a conditional probability density from which we derive confidence bounds on the future values of the mispricings and implement a trading decision process. We also comment on the research in course which will consider optimizing the perfomance of the decision process by using a Genetic Algorithm routine.

Keywords: Statistical arbitrage, algorithmic trading, neural networks, GARCH models.

1 Introduction

Statistical arbitrage consists in frequent trading on risk assets, such as stock, trying to long (buy the asset) when the price is at low levels and to short (to sell the asset) when the price is at high levels. Opposingly to deterministic arbitrage, which does the same with government bonds and other risk-free assets, statiscal arbitrage is riskier than deterministic arbitrage. In this article, we present an artificial intelligence algorithm based on statistical arbitrage to estimate the mispricing between a pair of stocks, and explore this mispricing to take a certain position in one of the two stocks in relation to time, with the goal to make a profit from the arbitrage.

The usual methodologies for the analysis by pairs trading are the correlation and cointegration, which search for identifying and testing short and long time relations between the assets, respectively. With the evolution of the studies in statistical arbitrage, the use of Artificial Neural Networks on the linear models has become more frequent and this is proposed in MEDEIROS et al.[6] for modelling interest tax. Following this line, another possible technique is to obtain the discrepancies between pairs of stocks and model them as a mean-reverting synthetic asset in order to describe its dynamics by means of time series analysis. In 2006, THOMAIDIS et al.[10] published an article proposing an intelligent tool combined with statistical methods to detect possible distortions on the prices of the pair of stocks by constructing the synthetic asset by an adaptive process and moving on to describe the dynamics of the distortions based on Artificial Intelligence together with some model describing the time-varying nature of the variance of the distortion's residuals. It is this methodology that we privilege in this work.

At first, we build a time series which corresponds to the mispricing between the two stocks. Afterwards, we utilize a neural network to estimate such mispricing on time t based on the $W = 300$ previous observations of the mispricing, utilizing a great percentage of our dataset to train the network. Following, we estimate GARCH parameters to estimate the standard deviation of the mispricing at each time, and check if our prediction implies on a change in the mispricing that is sufficiently large in comparison to such standard deviation. If it is, we consider this as an opportunity to take a short or long position (depending on the sign) to explore this price distortion. We still present improvements to the technique used, mainly on the decision process to take a (long or short) position.

2 Mispricing dynamics for a portfolio composed of two stocks

Let $\{P_{1,t}\}$ and $\{P_{2,t}\}$ be the time series of the prices of two cointegrated stocks, identified as 1 and 2, on the horizon $0 \leq t \leq T$. The standard procedure to identify statistical distortion on the pricings, or *mispricing*, is to run a regression of the values of one stock, say P_2, against the other (P_1, indeed) and to test the residues for zero-mean reversion as well as its stability along time, or (weak) *stationarity*. In this case, the mispricing is defined to be the residuals of this regression. Thus, given the assets represented by the time series $\{P_{1,t}\}$ and $\{P_{2,t}\}$, we define the *synthetic asset* constructed from them to be any time series of the form

$$\{a_{o,t} + a_{1,t}P_{1,t} + a_{2,t}P_{2,t}\} \quad , \tag{1}$$

$$\text{with mean-reversion } (0, \sigma_t^2), \quad \sigma_t^2 < \infty.$$

Each ordered pair $(a_{1,t}, a_{2,t}) \in \mathbb{R}^2$ represents the actual proportions of each asset to be kept on the operations portfolio, the negative sign of the coefficient meaning holding a *short position*, or "sell the corresponding asset", whereas the positive sign means hold-

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ing a *long position*, or “buy the corresponding asset”. In this way, the coefficients $a_{1,t}$ and $a_{2,t}$ give the ratio of each asset in an eventual positioning (a long/buy position in one asset and a short/sell position in the other one) on the synthetic asset at the instant t of the decision.

We follow the adaptive scheme in THOMAIDIS-KONDAKIS[9] in order to control the non-stationarity of the synthetic asset by which the coefficients are periodically re-estimated. Thus, we consider the “adaptive regression”

$$P_{2,t} = \alpha_{t-1} + \beta_{t-1} P_{1,t} + Z_t \ ,$$

where the time-varying coefficients α_t and β_t are given in the following manner. We compute the means in terms of a window chosen W,

$$M_{P_{1,2}} = mean\left\{\frac{P_{2,j}}{P_{1,j}};\ j = t-W+1, t-W+2, ..., t\right\} ,$$

$$M_{P_2} = mean\left\{P_{2,j} \ \ ; \ \ j = t-W+1, t-W+2, ..., t\right\} ,$$

and put

$$\beta_t = M_{P_{1,2}} \tag{2}$$

$$\alpha_t = (1-\beta_t) M_{P_2} \ \ . \tag{3}$$

Finally, we define the dynamics of the mispricings to be the time series $\{Z_t\}$ given by

$$Z_t = P_{2,t} - \alpha_{t-1} - \beta_{t-1} P_{1,t} \ \ , \tag{4}$$

for α_t and β_t as above. As we shall show in our validation studies in the sequel, the more frequent the values of the coefficients α_t and β_t are updated (that is, the bigger the value of W), the strongest is the mean-revertion, yielding more abrupt corrections for the mispricing.

3 Forecasting approach using an ANN-GARCH model

In high sampling frequencies, which is the case for intraday data, it turns out that the average uncertainty about the realised value, that is, the volatility of Z_t , is not constant over time but depends rather strongly on the history of Z_t. As for the prediction of the dynamics of the mispricings, we let $\hat{Z}_t$ be an estimate for Z_t modelled as

$$\hat{Z}_t = f(Z_{t-N}, Z_{t-(N-1)}, ..., Z_{t-2}, Z_{t-1}) \ , \tag{5}$$

where f is the output of a feedforward artificial neural network (ANN) with N inputs

$$(Z_{t-N}, Z_{t-(N-1)}, ..., Z_{t-2}, Z_{t-1})$$

and having a scalar as output which, once normalized, yields the price prediction. This is a 500 hidden layers ANN with each of them having 300 neurons. The activation function used for the training was the rectified linear function given by

$$f(x) = \max\{0, x\}.$$

In comparison with others activation functions such as $\arctan(x)$ or $\dfrac{1}{1+\exp(-x)}$, this particular one has the advantage of having low cost of computing. Note that, since this function is not differentiable on $x = 0$, one needs to approximate it by a continously differentiable function on a neighborhood of the origin (v. MAAS et al.[5] for details).

To forecast the mispricing volatility, we assume that the variance of the forecast $\hat{Z}_t$, conditional to $Z_{t-N}, Z_{t-(N-1)}, ..., Z_{t-2}, Z_{t-1}$, is time-varying and model such changes on the volatility using the GARCH (Generalized AutoRegressive Conditionally Heteroskedastic) developed in BOLLERSLEV[1]. In this kind of problems, we are concerned with modelling the return (i.e., the growth rate) of a time series.

For this reason, we introduce the new variables

$$R_t = \frac{Z_t - Z_{t-1}}{Z_{t-1}} \tag{6}$$

$$\hat{R}_t = \frac{\hat{Z}_t - \hat{Z}_{t-1}}{\hat{Z}_{t-1}} \ \ ,$$

as well as

$$R_t = \log Z_t - \log Z_{t-1} \tag{7}$$

$$\hat{R}_t = \log \hat{Z}_t - \log \hat{Z}_{t-1} \ .$$

We justify the sort of equivalence between (6) and (7) because if we apply the log function to

$$r_t = \frac{x_t - x_{t-1}}{x_{t-1}} \ ,$$

or, equivalently,

$$x_t = (1 + r_t) x_{t-1} \ ,$$

then

$$\log x_t - \log x_{t-1} = \log(1 + r_t)$$

and the Taylor series expansion results $\log(1+r_t) \approx r_t$, if r_t is a rather small percentage.

In general, the GARCH(m, s) methodology applied to an asset x_t models its return r_t as

$$r_t \quad = \quad \sigma_t \epsilon_t, \quad \text{where} \tag{8}$$

$$\sigma_t^2 \quad = \quad \alpha_0 + \sum_{i=1}^{m} \alpha_i r_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2, \tag{9}$$

with ϵ_t being a standard Gaussian white noise $N(0, 1)$,

In this study, we shall use the GARCH$(1, 1)$ model, with the ARCH library of the Python language providing the estimates for the values of α_0, α_1 and β_1 for the mispricings computed from our data base. Note that each σ_t can only be evaluated from r_{t-1} and σ_{t-1}.

In order to exploit the predictable component of the mispricing dynamics, we implement the decision criterion from the fact that the mispricing return is mean-reverting. Therefore, we take a fixed error margin α to build a reference confidence bound and compose it with the volatility σ_t obtained from the GARCH model to implement a trading strategy as follows:

- If $R_t - \hat{R}_t > (1-\alpha)\sigma_t$, we sell (short) the synthetic asset. This means that we sell 1 stock of P_2 and buy β_t stocks of P_1.
- If $R_t - \hat{R}_t < -(1-\alpha)\sigma_t$, we buy (long) the synthetic asset, that is, we buy 1 stock of P_2 and sell β_t stocks of P_1.

To obtain the contribuition of each trading positioning (shorting or longing) to the Profit and Loss (P&L) diagram of the arbitrage trading system, we assume that a positioning taken at t is realized (that is, the transaction is completed) the the following instant, $t+1$. Thus,

- Each longing taken at the instant t adds up $(Z_{t+1} - Z_t)$ to the P&L diagram.
- Each shorting taken at t adds up $-(Z_{t+1} - Z_t)$ to the P&L diagram.

4 Application to two stocks that are traded at Bovespa

We used as our data base the intraday prices, minute per minute, of the stocks PETR3 and PETR4 from 04/january/2016 to 02/may/2016, giving a total of approximately 35600 samples. We have taken PETR3 as the stock with price $P_{1,t}$ and PETR4 as the one with price $P_{2,t}$. Figure 1 ilustrates the history of the prices of the two stocks that were used wilth the purpose to validate our study.

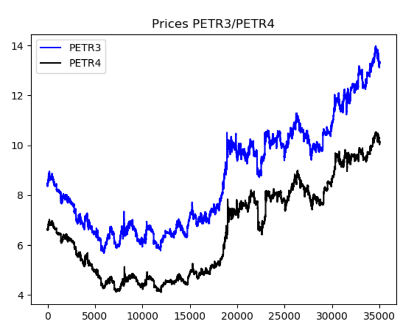

Figure 1. History of PETR3 and PETR4 from 04/01/2016 to 23/02/2016.
(Source: Bovespa).

After constructing the synthetic asset with the adaptive scheme for the coefficients, the time series of the mispricing $\{Z_t\}$ is shown in figure 2. For the ANN model $\{\hat{Z}_t\}$ given by (5) we used 80% of the mispricing series (approximately 0.80×35600 samples) for the training of our neural network and the remaining 1% of our data base were used to check the consistency of our trained network.

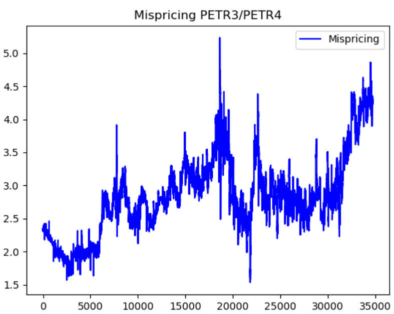

Figure 2. Mispricing obtained from the data base.

The graph in Figure 3 shows the ANN modelled $\{\hat{Z}_t\}$ in comparison with the historical mispricing $\{Z_t\}$. Note that the ANN estimative is substantially consistent.

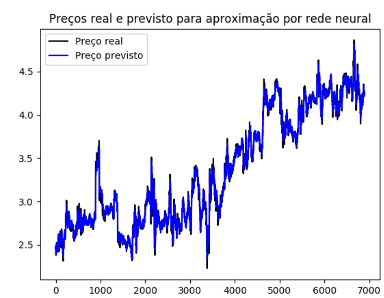

Figure 3. Z_t and $\hat{Z}_t$ compared.

Moreover, we can see in Figure 4 that the near coincidence of the peaks of the percentage variations (returns) of both Z_t and $\hat{Z}_t$ asserts the degree of precision of the estimative with neural network.

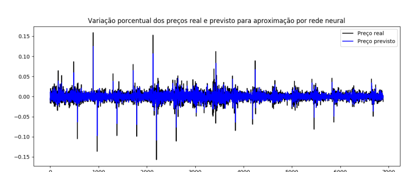

Figure 4. Comparison between the returns of the series for Z_t and $\hat{Z}_t$.

As for the forecast of the volatility, we used a GARCH$(1,1)$ model from the Python ARCH library to estimate the values of α_0, α_1 and β_1, resulting

$$\alpha_0 = 1.2483 \times 10^{-6} \quad , \quad \alpha_1 = 0.05 \quad , \quad \beta_1 = 0.93.$$

5 Discussion and further research

Figure 5 duplicates the realized synthetic asset $\{Z_t\}$ and, plotted in orange, the forecast $\{\widehat{Z}_{t+h}\}$ that was used as an auxiliary tool to determine the trading band according to the criterion establish above. In the graphs, $(1-\alpha) = 0.95$ is a confidence interval of a Gaussian distribution $N(0, \sigma_t^2)$ and σ_{t+h}^2 is the conditional variance *forecast* modelled by a GARCH$(1,1)$.

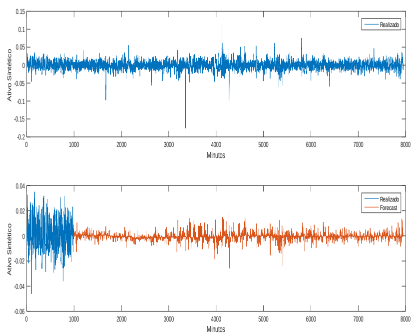

Figure 5. Realized synthetic asset and forecast *from* 04/01/2016 to 23/02/2016.

Considering a window of 20% of the data base and $(1-\alpha) = 0.95$ for the confidence bound, the resulting P&L diagram plotted in Figure 6 shows a substantial increase of the profit, which validates the perfomance of our arbitrage trading system:

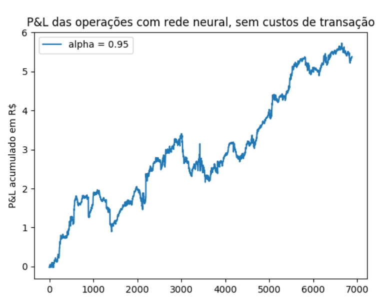

Figure 6. P&L of the arbitrage tradings.

The choice for $(1-\alpha) = 0.95$ is based upon the relation between the accumulated P&L and the confidence bound. As we infer from Figure 7, although the accumulated P&L turned out to be low for $(1-\alpha) = 0.95$, it is for this value that the average P&L is bigger and the number of tradings is minimal.

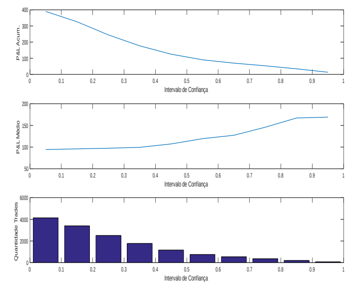

Figure 7. Trading strategy as a function of the confidence bound $(1-\alpha)\%$. (08/january/2016 to 21/january/2016)

As yet we have abstracted from the different realistic costs of transactions, bid-offer spread and so on. However, the present paper reports a research in the course of being done with deadline due to the end of 2018 and we shall consider these costs on the validation study together with further research aiming at an optimization of the trading system perfomance. In the sequel, we proceed to the sketching of the main line of the approach to be undertaken, which concerns tracking optimal bands using Genetic Algorithm. This strategy consists on using the Genetic Algorithm (GA) algorithm not exactly on the prediction of the time series but on an optimization process for building trading bands. Over the last decades, researchers from the time series prediction area have advanced some move towards using artificial intelligence techniques in their model constructions, including eventual gazes at the Nature as a source of inspiration. In particular, this is the case of GA for approaching problems of great complexity in numerical optimization. GA designates a family of computational procedures that dispense with any knowledge derived from the problem, except only for some form of assessment of the result, and in which a number of potential solutions for a problem paves the way for the evolution of a population. This is done with the following basic features:

(i) Each individual of a population is encoded from a solution as a string of symbols (genes). Thus, a genetic algorithm is based upon the encoding of the set of possible solutions (the population, and each of its individuals being the representation of a solution) and not upon the optimization parameters themselves. In our case, once the parameters are given by real values, the strategy consists in representing them directly on the string of symbols which encodes a solution through a RVR (real values representation) instead of binary codes from the alphabet $\{0,1\}$.

(ii) The measure of the quality of each solution is designated by means of a numerical value (fitness). This measure is set up by a so called *fitness function*

$$\begin{array}{cccc} J : & I \subset \mathbb{R}^N & \longrightarrow & \mathbb{R} \\ & v & \longmapsto & J(v) \end{array}$$

where I stands for the space (i.e., a set supplied with some algebraic structure) of individuals (solutions) of a population and the variable v stands for each individual.

(iii) New solutions (new individuals, new strings) are created by the application of the so called *genetic operators*. This process of creation of new individuals is made by genetic operators using transition rules that are probabilistic as opposed to deterministic. As a biological analogy, we can say that these new solutions constitute the offsprings and the solutions that are previous to the transition constitute the parents. In our case, the genetic operators that will be used are:

(1) Arithmetic crossover: each item/gene in the new string/solution (i.e., offspring) is a linear combination of the values in the previous string (parent) at the same positions.

(2) Mutation by Gaussian perturbation: this transition rule adds a value taken from a zero mean Gaussian distribution to a given item/gene.

(iv) The evolution of the process (i.e., the procedures in (iii) above) goes on until each individual/solution is sufficiently fit. The outcome of the algorithm is the best individual/string in the population according to the fitness function.

Specifically, the proposed genetic algorithm for this study is set up as follows:

Indivíduals. The population is made up of the individuals $v \in \mathbb{R}^2$ identified by isomorphism to the vectors

$$v = \begin{bmatrix} h \\ y \end{bmatrix} \in \mathbb{R}^{2\times 1},$$

where h is the time delay and $y = 1 - \alpha$ used in the construction of the trading band in our arbitrage system.

Initial population. The population starts off randomly with a constant number S of individuals $v_i = [h_i \quad y_i]^\top$, $i = 1, 2, \ldots, S$.

Determination of the fitness value of each individual. The fitness function that to be privileged at first is the cost functional $J : I \subset \mathbb{R}^{2\times 1} \to \mathbb{R}$ defined by

$$J(v) := \int_{t_o}^{t_h} \left(P\&L_R(t) - P\&L_v(t)\right)^2 dt \quad , \quad \forall\, v \in I, \tag{10}$$

where
(i) $P\&L_R$ is the accumulated P&L chosen as the reference object for the tracking. For instance, the $P\&L_R$ can be obtained by an idealized succession of tradings done when one knows completely of the realized time series $\{Z_t\}$ over the past horizon. For the purpose of this work, we shalll consider as the reference $P\&L_R$ the accumulated P&L for tradings without transaction costs.
(ii) $P\&L_v$ is the accumulated P&L obtained by a succession of tradings done within the realized time series $\{Z_t\}$ over the forecasted horizon using the trading band built in association with v.

The fitness value of each individual is computed by using the cost functional (10) so that the minimization of $J(v)$ is equivalent to the maximization of the fitness values. Once the population is taken to be made up of individuals $[h_i \quad y_i]^\top$ distributed according to their fitness values, the greatest lower bound for the values of the cost functional corresponds to the highest position of the population vector, that is, the string v^* such that $J(v^*) = \inf\{J(v)\ ; v \in I\}$.

Crossover. Once the initialization and the determination of the fitness value are completed, the operation of crossover is made by replacing the genetic codes between pairs of individuals. Let $a = [a_1 \quad a_2]^\top$ and $b = [b_1 \quad b_2]^\top$ be two offsprings and $z = [z_1 \quad z_2]^\top$ and $w = [w_1 \quad w_2]^\top$ be two individuals among the parents. Then, the crossover is done according to the following transition rule:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \lambda \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + (1-\lambda) \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

and

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \lambda \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + (1-\lambda) \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} ,$$

where λ is some random number with normal distribution in $[0, 1]$.

Mutation. Each individual originated during the crossover is perturbed with a given probability (for instance, 0.2) within a variation scale δ fixed according to the formula

$$\begin{bmatrix} h_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} h_i \\ y_i \end{bmatrix} + \delta \begin{bmatrix} 1 - 2\xi_1 \\ 1 - 2\xi_2 \end{bmatrix} ,$$

where each ξ_i is a random number in the interval $[0, 1]$.

New generation. In the subsequent stage, a subset consisting of old individuals as well as those created in the crossover stage is removed from the set of the total population by cutting off the elements that violate certain restriction (yet to be established by us) and distributed and sorted out in accord with the fitness value. The next generation should contain $\frac{2}{3}S$ of the best elements. In order to prevent from the eventuality of being stuck in local optimal points, $\frac{1}{3}S$ of the individuals are drawn out again through a normal distribuition in $[0, 1]$.

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